

Hadron–Resonance Gas at Freeze–out: Reminder on Importance of Repulsive Interactions

V.V. Begun,^{1,2} M. Gaździcki,^{2,3} and M.I. Gorenstein^{1,4}

¹*Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*

²*Goethe–University, Frankfurt, Germany*

³*Jan Kochanowski University, Kielce, Poland*

⁴*Frankfurt Institute for Advanced Studies, Frankfurt, Germany*

Abstract

Strong interactions between hadrons include attractive and repulsive components. In statistical approaches attractive interactions are modelled by including resonances while repulsive interactions by taking into account a finite proper volume of particles. In this paper the influence of repulsive interactions on matter properties is considered within the excluded volume van der Waals hadron–resonance gas. Quantitative results are presented for matter at the chemical freeze–out in central nucleus-nucleus collisions at relativistic energies. In particular, it is shown that repulsive interactions lead to a significant decrease of collision energy at which the net–baryon density has a maximum.

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I. INTRODUCTION

Statistical models of the hadron gas are an important tool to extract properties of matter created in relativistic nucleus-nucleus collisions (see, e.g., Refs. [1–7]). Basic parameters of these models are the matter temperature T , baryon chemical potential μ_B , and volume V . If supplemented by additional model parameters monitoring deviations from the chemical equilibrium [8], they approximately fit rich data on mean hadron multiplicities in a broad range of reactions, from e^+e^- , $p+p$, and $p+\bar{p}$ [9] at low energies to central Pb+Pb collisions at the highest LHC energy [10].

The most popular version of the statistical models of hadron matter is the ideal hadron–resonance gas (I–HRG), i.e., a statistical system of non-interacting hadrons and resonances. One argues, based on the Dashen, Ma and Bernstein theorem [11], that resonances introduced to the ideal hadron gas take into account attractive interactions between hadrons. The repulsive part of interactions between hadrons is usually accounted for by the van der Waals excluded volume procedure generalized to the relativistic case of a variable number of hadrons [12]. The resulting excluded volume model is not anymore an ideal gas model, and in this paper it will be denoted as the EV–HRG model. Another popular example of modelling attractive and repulsive interactions between hadrons is the relativistic mean field theory in a form of the Walecka model [13] and its different modifications (see e.g., recent paper [14] and references therein). In this approach, scalar and vector meson fields describe respectively the attractive and repulsive forces between baryons.

Both attractive and repulsive interactions are important for qualitative as well as quantitative description of properties of strongly interacting matter. For example, the nucleon-nucleon potential includes both parts – attractive at large and repulsive at small distances. The presence of both attractive and repulsive interactions between nucleons is crucial for the existence of stable nuclei. Moreover an important undesired feature of the I–HRG model at high temperatures was noted in Refs. [15, 16]. Due to the large number of different types of baryons and mesons, the point-like hadrons would always become the dominant phase at very high energy density. Just the excluded volume effects ensure a transition from a gas of hadrons and resonances to the quark–gluon plasma.

The aim of this paper is to recall the role of repulsive interactions between hadrons. It is shown that the excluded volume hadron–resonance gas model yields different properties of matter at the freeze–out than the ideal hadron–resonance gas, if densities and their collision energy dependence are considered. Quantitative results are presented for two examples, namely collision energy dependence of the net–baryon density [17] as well as the ratio of baryon and meson entropy density [18–20]. They are selected, because of conjectures that the maximum baryon density as well as the transition between baryon and meson dominated matter may be related to the onset of deconfinement observed in central Pb+Pb collisions at the CERN SPS energies [21]. Note that chemical freeze-out parameters T and μ_B in nucleus–nucleus collisions are straightforwardly connected to the data on hadron multiplicities. The energy range considered in this paper is presently studied experimentally at the CERN SPS [22] and the BNL RHIC [23]. In future this effort will be extended by experiments at new accelerators, at the JINR NICA [24] and the FAIR SIS–100 [25, 26].

The paper is organized as follows. In Sec. II the ideal hadron–resonance gas model is introduced. The excluded volume hadron–resonance gas model is presented in Sec. III, where also quantitative results for densities at the chemical freeze–out in central Pb+Pb collisions are given and discussed. A summary given in Sec. IV closes the paper.

II. IDEAL HADRON–RESONANCE GAS

In the grand canonical ensemble the pressure of the I–HRG is given by

$$p^{id} = \sum_i p_i^{id}(T, \mu_i) = \sum_i \frac{d_i}{6\pi^2} \int_0^\infty \frac{k^4 dk}{(k^2 + m_i^2)^{1/2}} \left[\exp\left(\frac{\sqrt{k^2 + m_i^2} - \mu_i}{T}\right) + \eta \right]^{-1}, \quad (1)$$

where T is the system temperature, $\eta = -1$ and $\eta = 1$ for bosons and fermions, respectively, while $\eta = 0$ corresponds to the Boltzmann approximation. For a hadron i , m_i is its mass and d_i is the spin degeneracy. The chemical potential is given by

$$\mu_i = b_i \mu_B + s_i \mu_S + q_i \mu_Q \quad (2)$$

with $b_i = 0, \pm 1$, $s_i = 0, \mp 1, \mp 2, \mp 3$ and $q_i = 0, \pm 1, \pm 2$ for hadrons. The number density of a hadron i reads:

$$n_i^{id}(T, \mu_i) = T \frac{\partial p^{id}}{\partial \mu_i} = \frac{d_i}{2\pi^2} \int_0^\infty k^2 dk \left[\exp\left(\frac{\sqrt{k^2 + m_i^2} - \mu_i}{T}\right) \pm 1 \right]^{-1}. \quad (3)$$

Considering the temperature T , baryon chemical potential μ_B , and volume V as free parameters one can fit mean hadron multiplicities $\langle N_i \rangle = V n_i$ measured in relativistic nucleus–nucleus collisions at each collision energy. In this analysis, μ_S and μ_Q are expressed as functions of T and μ_B when the conditions on strangeness, $\langle S \rangle = 0$, and electric to baryon charge ratio, $\langle Q \rangle / \langle B \rangle = Z/A$, are taken into account. Most of experimental data on nucleus–nucleus collisions concern yields of long–lived hadrons, which include products of resonance decays. This requires a proper treatment of short lived resonances, namely products of their strong and electromagnetic decays should be added to mean multiplicities of stable hadrons. In this paper the numerical implementation of the hadron–resonance gas model provided by the THERMUS package [27] is used to calculate the relevant quantities according to Eqs. (4-6). Particles and resonances (all mesons up to $K_4^*(2045)$) and baryons (up to Ω^-), quantum statistics as well as width of resonances are included.

The analysis of central Pb+Pb (Au+Au) collisions registered by experiments at SIS, AGS, SPS, and RHIC allows to establish the collision energy dependence of T and μ_B which can be parameterized as [19]:

$$T = 166 - 139 \mu_B^2 - 53 \mu_B^4, \quad \mu_B = \frac{1308}{1000 + 0.273 \sqrt{s_{NN}}}, \quad (4)$$

where $\sqrt{s_{NN}}$ is the center of mass energy of a nucleon pair. Values of T , μ_B , and $\sqrt{s_{NN}}$ in Eq. (4) should be given in MeV. The chemical freeze–out line, $T = T(\mu_B)$, as well as the energy dependence of the T and μ_B parameters are shown in Fig. 1 (a) and (b), respectively.

The net–baryon density ρ_B , entropy density s and energy density ε can be found from the system pressure p using the thermodynamical relations:

$$\rho_B = \frac{\partial p}{\partial \mu_B}, \quad s = \frac{\partial p}{\partial T}, \quad \varepsilon = T \frac{\partial p}{\partial T} + \mu \frac{\partial p}{\partial \mu} - p. \quad (5)$$

With chemical freeze–out parameters given by Eq. (4) and the ideal gas expression Eq. (1) for the system pressure, one finds the quantities in Eq. (5) as functions of collision energy.

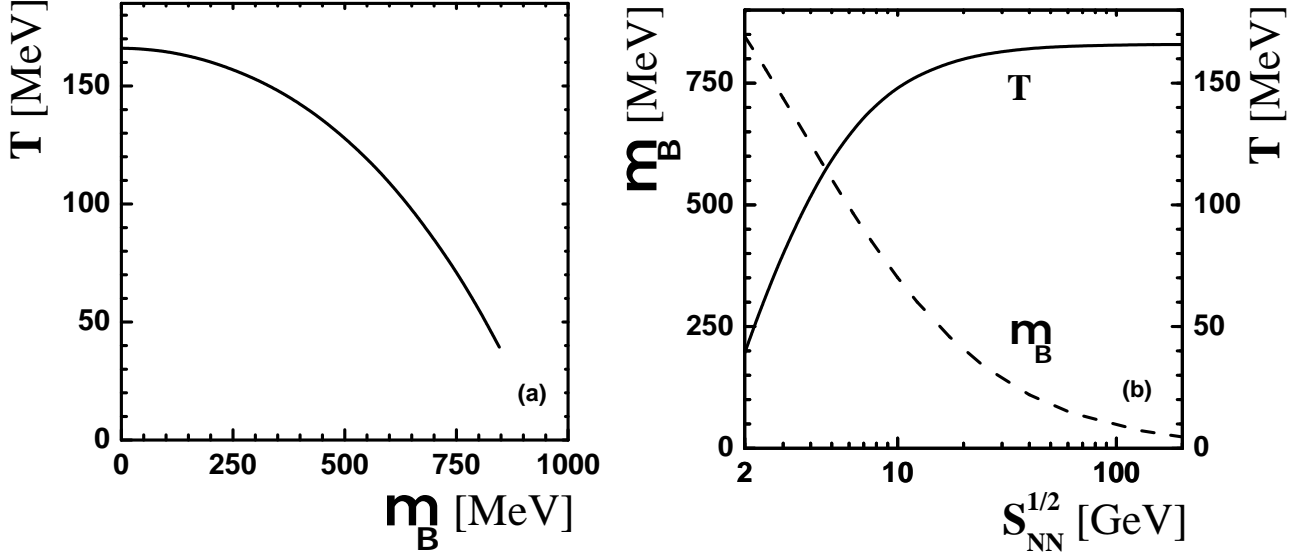


FIG. 1: (a): The chemical freeze-out line $T = T(\mu_B)$. (b): The T and μ_B along the chemical freeze-out as function of $\sqrt{s_{NN}}$.

The I-HRG model is based on the assumption of complete thermal and chemical equilibrium. An additional I-HRG parameter, the strangeness suppression factor γ_S , has to be introduced to account for deviations of strange hadron multiplicities from chemical equilibrium [8]. Its dependence on T and μ_B obtained by fitting hadron yields measured in the full phase-space can be parameterized as [7]

$$\gamma_S = 1 - 0.396 \exp\left(-1.23 \frac{T}{\mu_B}\right). \quad (6)$$

At the AGS and SPS energies the γ_S parameter is significantly smaller than one, which means under-saturation of strange hadron yields with respect to the chemical equilibrium. The γ_S parameter should be included in the model if a proper description of strange hadron yields is required. However, the relation between T and μ_B as well as the chemical freeze-out line Eq. (4), obtained within the models with and without the γ_S parameter is similar.

The baryon density ρ_B^{id} as a function of collision energy calculated following the freeze-out line (4) is shown in Fig. 2 (a). In this and the following figures the laboratory collision energy per projectile nucleon E_{lab} is used to present the dependence on collision energy. Its connection to center-of-mass energy per nucleon pair, $\sqrt{s_{NN}}$, is given by $\sqrt{s_{NN}} = \sqrt{2m_N E_{lab} + 2m_N^2}$,

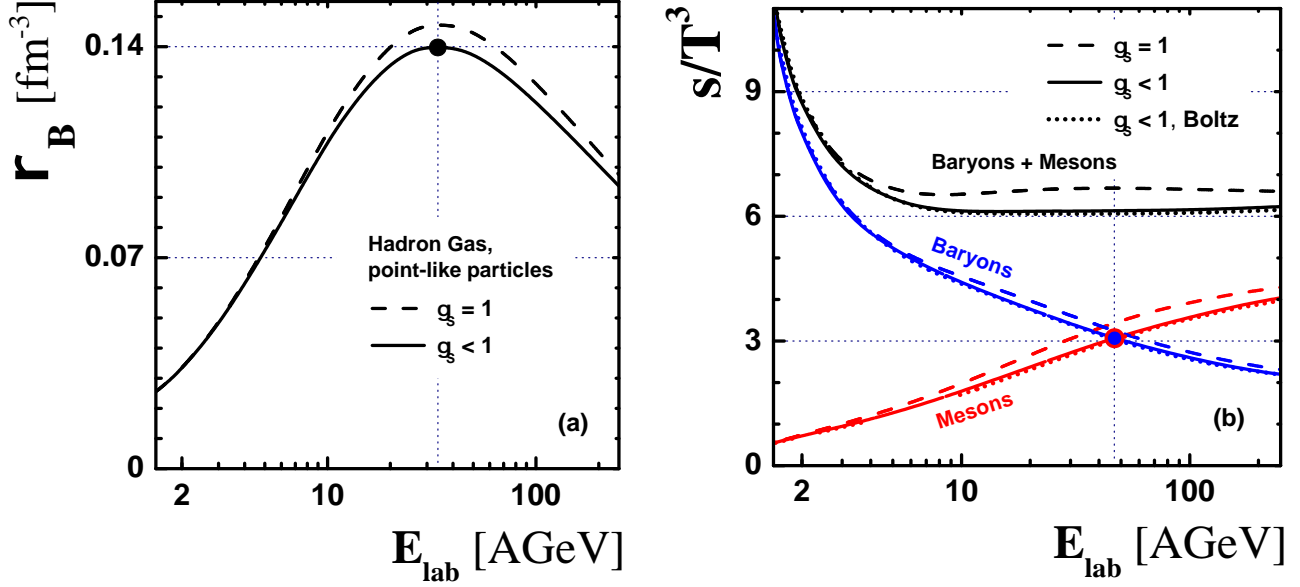


FIG. 2: The net-baryon density ρ_B (a) and the ratio s/T^3 (b) along the chemical freeze-out line Eq. (4) and γ_S according to Eq. (6) are shown by the solid lines. The dashed lines correspond to $\gamma_S = 1$ for ρ_B (a) and s/T^3 (b). The dotted line corresponds to the Boltzmann approximation in s/T^3 .

where m_N is the nucleon mass. As seen in Fig. 2 (a), the baryon density has a maximum [17] at $E_{lab} \cong 34A$ GeV. This is the collision energy at which the NA49 Collaboration observed the maximum of the K^+/π^+ ratio (the *horn*) and other signals of the onset of deconfinement [21].

The total entropy density as a function of collision energy following the freeze-out line Eq. (4) is shown in Fig. 2 (b). Meson s_M and baryon s_B entropy densities are also presented in the figure. With increasing collision energy the baryon-dominated ($s_B > s_M$) matter changes to meson-dominated ($s_M > s_B$) matter. For the I-HRG model this transition is located at $E_{lab} \cong 46A$ GeV.

For the $T-\mu_B$ values at the chemical freeze-out line Eq. (4) the role of quantum statistics is small. For baryons the Fermi statistics changes their densities by less than 1%. The largest density change due to the Bose statistics is for pions. It is, however, still smaller than 10%. The ratio s/T^3 calculated within the Boltzmann approximation, i.e. $\eta = 0$ in Eq. (1), is shown in Fig. 2 (b) by the dotted line. The deviations from the results with quantum statistics included

are hardly visible. They are even smaller for ρ_B and thus the corresponding line calculated with the quantum statistics included is not plotted. The collision energy dependence of ρ_B and s/T^3 calculated for the γ_S parametrization Eq. (6) and for $\gamma_S = 1$ is also shown in Fig. 2. One concludes that the energy at which ρ_B has the maximum as well as the energy of the transition between baryon-dominated and meson-dominated matter are approximately independent of the quantum statistics and the degree of strangeness equilibration.

It was suggested [18] that the maximum of net-baryon density and/or the transition from baryon to meson dominance may be related to the anomalous behavior of the K^+/π^+ ratio [21]. In the next section these phenomena are examined by taking into account repulsive interactions between hadrons.

III. EXCLUDED VOLUME HADRON-RESONANCE GAS

The results presented in Section II have been obtained within the ideal hadron-resonance gas model in which only attractive interactions between hadrons are taken into account by the inclusion of resonances. In this section the role of repulsive interactions is considered within the excluded volume hadron-resonance gas model.

The van der Waals excluded volume procedure corresponds to a substitution of the system volume V by the available volume $V - \sum_i v_i N_i$, where $v_i = 4(4/3\pi r_i^3)$ is the excluded volume parameter and r_i is the corresponding hard sphere radius of a particle i . This result, in particular, the presence of a factor of 4 in the expression for v_i , can be rigorously obtained for a low density gas of particles of a single type (see, e.g., Ref. [28]). In the grand canonical ensemble, the substitution $V \rightarrow V - \sum_i v_i N_i$ leads to a transcendental equation for the pressure of the EV-HRG¹ [12, 15]:

$$p = \sum_i p_i^{id}(T, \tilde{\mu}_i) ; \quad \tilde{\mu}_i = \mu_i - v_i p . \quad (7)$$

Using Eq. (5) one finds the net-baryon, entropy and energy densities:

$$\rho_B = \frac{\sum_i b_i n_i^{id}(T, \tilde{\mu}_i)}{1 + \sum_j v_j n_j^{id}(T, \tilde{\mu}_j)} , \quad s = \frac{\sum_i s_i^{id}(T, \tilde{\mu}_i)}{1 + \sum_j v_j n_j^{id}(T, \tilde{\mu}_j)} , \quad \varepsilon = \frac{\sum_i \varepsilon_i^{id}(T, \tilde{\mu}_i)}{1 + \sum_j v_j n_j^{id}(T, \tilde{\mu}_j)} . \quad (8)$$

¹ A discussion of other excluded volume formulations can be found in Ref. [29].

In comparison to the corresponding densities calculated within the I-HRG model the densities in the EV-HRG model Eq. (8) are lower because of two reasons:

- i) due to the suppression factor $[1 + \sum_j v_j n_j^{id}(T, \tilde{\mu}_j)]^{-1}$ and
- ii) due to the shift in chemical potential $\mu_i \rightarrow \tilde{\mu}_i$ which in the Boltzmann approximation leads to the suppression factor $\exp(-v_i p/T) < 1$.

The shift of the chemical potential makes the Boltzmann approximation even more accurate than in the case of the ideal gas. If all proper volume parameters are the same $v_i = v$ (i.e. $r_i = r$), the Boltzmann approximation gives the total suppression factor R

$$R(T, \mu_B; r) = \frac{\exp(-v p/T)}{1 + v \sum_j n_j^{id}(T, \tilde{\mu}_j)} , \quad (9)$$

the same for all densities of Eq. (8):

$$\rho_B(T, \mu_B) = R \rho_B^{id}(T, \mu_B) , \quad s(T, \mu_B) = R s^{id}(T, \mu_B) , \quad \varepsilon(T, \mu_B) = R \varepsilon^{id}(T, \mu_B) \quad (10)$$

and $n_i(T, \mu_B) = R n_i^{id}(T, \mu_B)$. Typical values of hard-core radii considered in the literature [2, 30] are $r = (0.3 \div 0.8)$ fm. The energy dependence of the suppression factor Eq. (9) calculated along the chemical freeze-out line for $r = 0.5$ fm and $r = 1$ fm is shown in Fig. 3. The R factor (9) decreases monotonously with increasing collision energy. For example, for $r = 0.5$ fm one finds $R \cong 0.9$ and $R \cong 0.4$ at small and large E_{lab} , respectively. One may therefore expect a decrease of the value of ρ_B at its maximum by a factor of 0.5 (for $r = 0.5$ fm), and a shift of the position of the maximum to a smaller collision energy. In fact, in Fig. 4 (a) one observes that the maximum of the net-baryon density is located at $E_{lab} \cong 17A$ GeV for $r = 0.5$ fm and at $E_{lab} \cong 7A$ GeV for $r = 1$ fm, instead of $E_{lab} \cong 34A$ GeV for the I-HRG model. It is also seen that the value of ρ_B at the maximum decreases strongly with increasing value of the hard-core radius. The entropy density shown in Fig. 4 (b) is reduced by the same suppression factor. The collision energy at which baryon and meson entropy densities are equal is, however, independent of R and is located at $E_{lab} \cong 46A$ GeV. This is true only if the hard-core radius parameter is the same for all hadrons.

It is instructive to consider the role of the excluded volume effects for different hard-core radii of baryons r_B and mesons r_M . As an example, the results for $r_B = 0.5$ fm and $r_M = 0$ are presented in Fig. 4. In a comparison to the results for $r = 0.5$ fm for all hadrons one observes

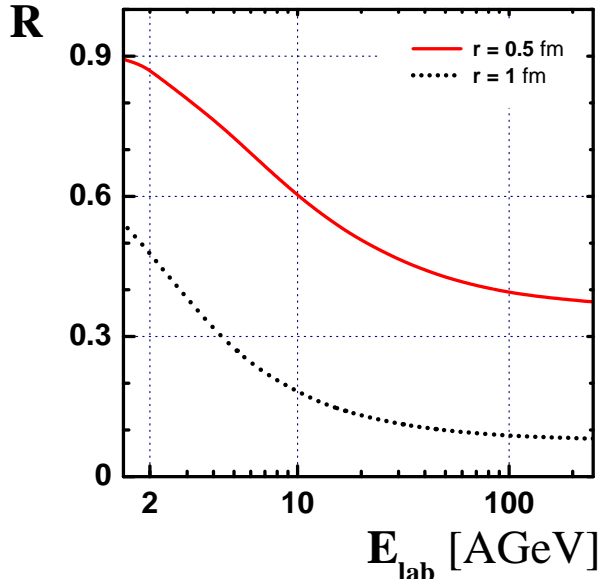


FIG. 3: The excluded volume suppression factor R Eq. (9) as a function of E_{lab} along the chemical freeze-out line Eqs. (4,6). The solid and dotted lines correspond to $r = 0.5$ fm and $r = 1$ fm, respectively.

small changes of ρ_B but a significant shift of the transition point between baryon and meson dominated matter. Its position decreases from $E_{lab} \cong 46A$ GeV to $E_{lab} \cong 23A$ GeV. The model with non-equal radii of hadrons needs, however, further detailed studies. This is because fits to the hadron yields performed with the EV-HRG model with non-equal radii should give different freeze-out parameters T and μ_B than those in Eq. (4) obtained within the I-HRG model. This analysis is beyond the scope of the paper.

IV. SUMMARY

The ideal hadron-resonance gas model in the grand canonical ensemble is simple and has only few free parameters. In spite of this it is successful in describing the bulk properties of mean hadron multiplicities in high energy collisions. The model takes into account attractive interactions between hadrons but ignores repulsive interactions. The latter are needed to catch basic qualitative features of strong interactions, e.g, the phase transition between hadron-

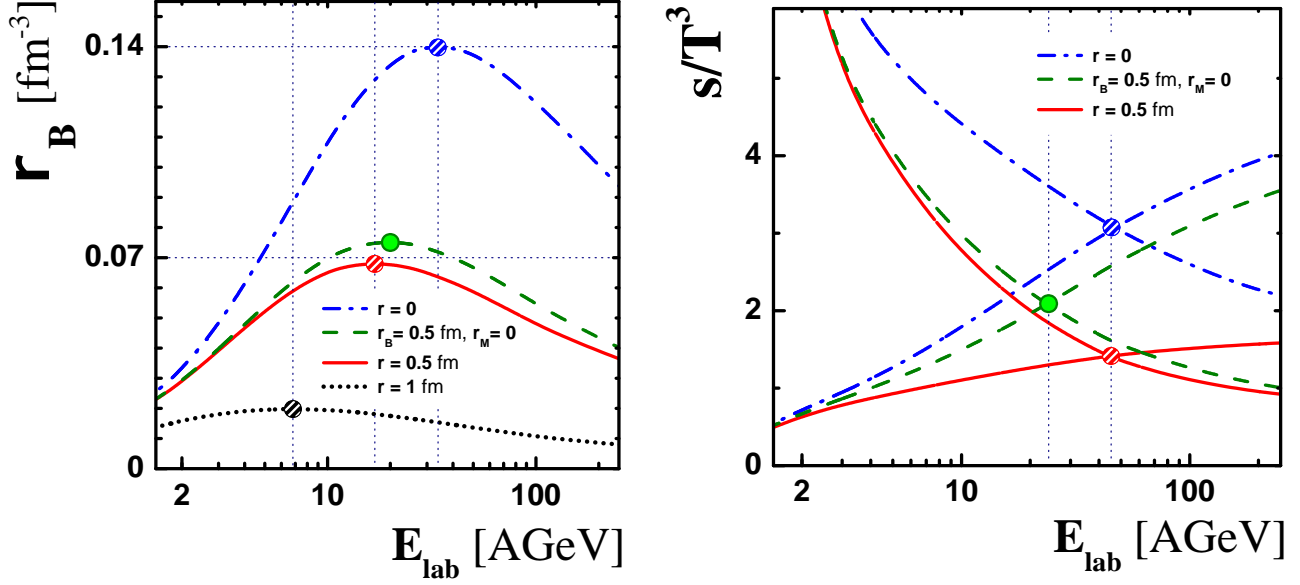


FIG. 4: (a): The baryon density along the chemical freeze-out line from Eqs. (4,6). Dashed-dotted line corresponds to the model with $r = 0$, dashed line to $r_B = 0.5 \text{ fm}$ and $r_M = 0$, solid line to $r_B = r_M = r = 0.5 \text{ fm}$, and dotted line to $r_B = r_M = r = 1 \text{ fm}$. (b) The ratios s_B/T^3 and s_M/T^3 along the chemical freeze-out line Eqs. (4,6). Dashed-dotted lines corresponds to the model with $r = 0$, solid lines to $r_B = r_M = r = 0.5 \text{ fm}$, and dashed-dotted lines to $r_B = 0.5 \text{ fm}$ and $r_M = 0$.

resonance gas and the quark-gluon plasma. Moreover, they strongly modify properties of the hadron-resonance gas. The most common way to include repulsive interactions in the hadron-resonance gas model is to follow the van der Waals excluded volume procedure and introduce hard-core radii of hadrons.

If radii of all hadrons are assumed to be the same, the chemical freeze-out parameters, temperature and baryon chemical potential, fitted to data on mean hadron multiplicities are identical to those obtained within the ideal hadron-resonance gas model. However, all densities calculated within the van der Waals model are lower than the corresponding densities obtained within the ideal gas model and thus the fitted volume parameter in the van der Waals gas formulation is significantly larger. The density suppression factor R depends on the T and μ_B parameters, which in turn depend on collision energy. Consequently the collision energy dependence of densities is sensitive to the assumed hard-core radius of hadrons. In particular,

the energy at which net-baryon density has a maximum decreases from about $E_{lab} \cong 34A$ GeV for the ideal gas model to about $E_{lab} \cong 7A$ GeV for the excluded volume model with $r = 1$ fm.

If the radii of hadrons are assumed to be different, densities of different hadrons are modified differently. In particular, the collision energy at which the transition between baryon and meson dominated matter takes place decreases from $E_{lab} \cong 46A$ GeV for the ideal gas model to $E_{lab} \cong 23A$ GeV for the excluded volume model with the hard-core radii of baryons and mesons equal to 0.5 fm and 0, respectively. Clearly, the excluded volume effects are even larger for the hadron matter at stages preceding the chemical freeze-out in nucleus-nucleus collisions, i.e. at larger values of the energy density.

In view of this study, the suggestions that the collision energy of the onset of deconfinement coincides with the energy at which the net-baryon density at the chemical freeze-out reaches its maximum and/or the transition between baryon and meson dominated matter takes place are premature. This is because the results obtained within the hadron gas approach are strongly dependent on the modelling of repulsive interactions between hadrons. Experimental and/or theoretical methods to better estimate hard-core radii of hadrons within the excluded volume model are needed to improve our understanding of the properties of hadron-resonance matter.

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